OPTIMAL DESIGN OF REINFORCED CONCRETE COUNTERFORT RETAINING WALLS

Pro.Dr. NabeelAbdulrazzaqJasim

FalahMajeedHameed

ABSTRACT

Mathematical programming techniques have been used to minimize the cost of reinforced concrete counterfort retaining wall.The study presents a formulation based on elastic analysis and the ultimate strength method of design as per ACI-M318code. A computer program is generated to handle the considered problem. The formulation of optimization problem has been made by utilizing the interior penalty function method as an optimization method with the purpose of minimizing the objective function representing the cost of one-meter length of the counterfort retaining wall. This includes cost of concrete, reinforcement, and formwork. The design variables considered in this study are the dimensions and the amounts of reinforcement.

It is found that the optimal spacing of counterforts equals about (0.214 to 0.366) of total height of wall. The optimum width of the base is found in the range (0.50 to 0.78) of the total height of the wall. Also the thickness of the stem is in the range(0.0284 to 0.0377) of the total height and it is less than half thickness of the base.

Keywords: optimization,penalty function,reinforced concrete, Counterfortretainingwalls

التصميم الأمثل للجدران الساندة للتربة من الخرسانة المسلحة والمدعمة بأجنحة

تم استخدام تقنيات رياضية برمجية لتصميم جدار خرساني ساند مدعم بأجنحة بأقل كلفة. واعتمدت الدراسة التحليل المرن للمنشأ والتصميم بطريقة المقاومة القصوى وفقا لمتطلبات نظام التصميم الأمريكي (ACI- Code). وابتكر برنامج لمعالجة مسالة البحث. وفي صياغة مسالة الأمثلية فقد تم الاستفادة من طريقة دالة الجزاء(penalty function method)للحصول على القيمة الصغرىلدالة الهدف وهي كلفة متر واحد من طول الجدار الساند0 وهذه الكلفة تتضمن كلفة الخرسانة وحديد التسليح وأعمال القالب.أن المتغيرات التصميمية المعتمدة في هذه الدراسةهيإبعاد وكميات حديدالتسليح .

ولقد وجد بان المسافة المثالية بين الأجنحة هي (0.214الى 0.366) من الارتفاع الكلي للجدار وان العرض المثالي للقاعدة يتراوح بين (0.50الى 0.78) من الارتفاع الكلي للجدار. وكذلك وجد بان سمك الجدار يتراوح بين (0.0377الى 0.0284) من الارتفاع الكلي وهو اقل من سمك القاعدة.

INTRODUCTION

If retaining walls having height of filling more than (6m), is designed as cantilever type retaining wall, the thickness of stem wall becomes excessive and design will be uneconomical [1]. Such walls should be designed as counterfort type retaining walls.

Analysis of a counterfort retaining wall proceeds with the selection of provisional dimensions for the retaining wall, which are then analyzed for stability and other structural requirements, and subsequently revised, if required. Since this is a trial-and-error process, several solutions to the problem may be possible. Many of these solutions may be structurally satisfactory, but need not necessarily be so from the economic point of view.

Several authors have surveyed the utilization of optimization in structural design.Chou [2] (1977) studied the optimum design of reinforced concrete T-beam sections.The Lagrange multipliers technique was used to solve the problem.Subramanyamand Adidam[3](1981)used the limit state method and mathematical programming to get the optimal designs of typical T-beam floor. The interior penalty method was utilized to get the solution. A comprehensive method of finding out the optimum cross-section of a reinforced concrete cantilever retaining wall has been discussed elsewhere briefly by Choundhury [4] in 1980. Ibrahim [5] (1999) developed a computer program for the optimum design of T-beam floor based on ACI-318-89 Code requirements for both ultimate and serviceability limit states constraints.The interior penalty method was used.

In this study an attempt is made to obtain an economical design which satisfies building code requirements for reinforced concrete ACI 318M-2005 code. A mathematical programming method based on the concepts of the ultimate strengththeory and an optimization technique is developed.

Formulation of The Problem

1) Design variables

In the design procedure of the counterfort retaining wall, some parameters are considered to be constant along the designprocesses,and theyshould be givenat the start of the program. These include:-

1. Soil parameters ϕ and $c$for both backfill and base soil (ϕ_1,ϕ_2, c_1 and c_2).
2. Height (H_2) of counterfort retaining wall
3. The bearing capacity of soil.
4. Unit weight of soils (γ_{s1}andγ_{s2}), concrete (γ_c), and steel γ_{steel}.
5. The minimum cover for the reinforcement of the stem and base.
6. The compressive strength of the concrete (f'_c) and the yield strength of the steel (f_y).
7. The ratio (R_1) of the cost of concrete per cubic meter to cost of reinforcement per Newton, and the ratio (R_2) of the cost of formwork per square meter to cost of reinforcement per Newton.

The design variables are thegeometric dimensions and the different steel reinforcement areas [Fig. (1)].The geometricdimensions include: D_s thickness of stem; D_b thickness of base; B width of base ($B=L_t+ D_s +L_h$); L_c distance between counterforts center to center. While the steel reinforcement includes:

a) A_{s1}: the steelarea of main reinforcement at the bottom of toe.

b) $A_{s2}$and A_{s4}: the area of shrinkage and temperature steel reinforcement at the bottom and top of the toe in longitudinal direction.

c) A_{s3}: the area of shrinkage and temperature steel reinforcement at the top of toe

d) A_{s5}: the steel area of main reinforcement at the top of heel.

e) A_{s6}: the steel area of the reinforcement at the top of heel in longitudinal direction.

f) A_{s7}: the steel area of shrinkage and temperature reinforcement at the bottom of heel.

g) A_{s8}: the steel area of reinforcement at the bottom of heel in longitudinal direction.

h) $A_{s9}$and A_{s10}: the steel area of horizontal reinforcement at the stem in the two faces.

i) $A_{s11}$and As_{12}: the steel area of vertical reinforcement at the stem in the two faces.

j) A_{s13}: the steel area of reinforcement at the counterfort.

k) As_{14} and As_{15}: the tension steel to tie counterfort to the stem and the base, respectively.

In this study the followings are used:

1- ϕ_1=30and c_1=0 or backfilland ϕ_2=28°and c_2=1912 N\m^2 for the base soil.

2- The Rankine earth pressure coefficientsK_a(0.361 ,0.333)and K_p(2.8, 3) are used.

3- The load factor LF= 1.7.

4- The trial dimensions are chosen using Fig. (2) as a guide.

5- The stem thickness (Ds) based onwidebeam shearby takingthe criticalsectionat the base slab junction.This thickness is

assumed to be constant along the stem.

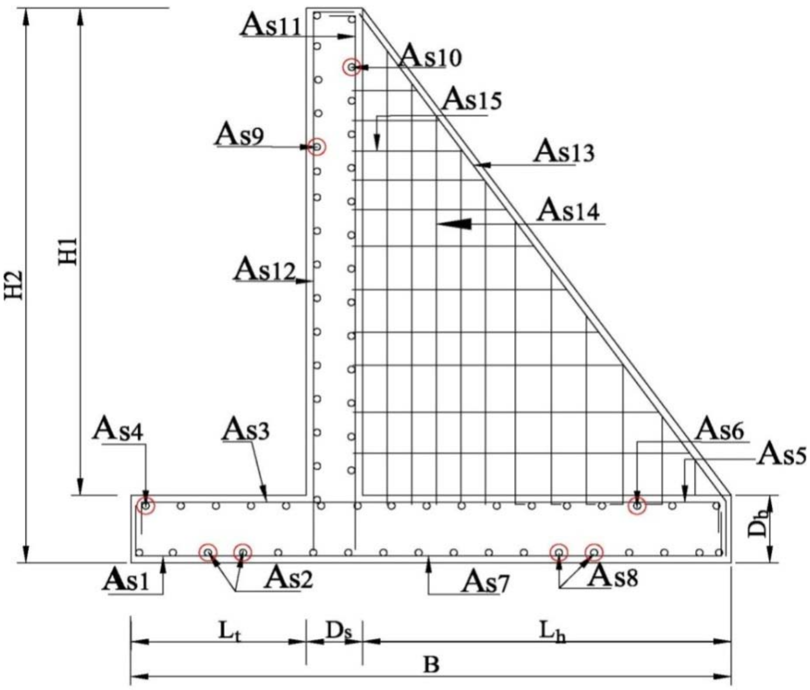


Fig. (1) reinforcement of counterfort retaining wall

6- Wall stability for overturning and sliding are checked and the resultant R is in the middle third of the base B, i.e., the eccentricity should be ($e \leq L/6$).

2) Analysis of Structure

Counterfort retaining walls are indeterminate problems which can be solved using plate theory [6]. Simplified methods are commonly used to solve the problem [6].Huntington's design procedure is usedin this study and shown in Figs. (2), (3) and (4).

3) DesignConstraints.

The design is required to satisfy two groups of constraints namely, the general constraints and the ultimate strength requirements in accordance with ACI-318M-2005 code. The explanations of these constraints are given below.

a) The General Constraints

These constraints relate to the general stability of the retaining wall and the soil resistance, and include:

1. Overturning:

$$\frac{resisting\ moment}{overturing\ momet} \geq F_o$$

or

$$\frac{M_o}{M_r} = F_o \text{---------}(1)$$

where:

F_o= Factor of safety against overturning

2. Sliding:

$$\frac{\text{resisting force}}{\text{overturning force}} \geq F_s$$

or

$$\frac{P_r}{P_{a2}} \text{---------}(2)$$

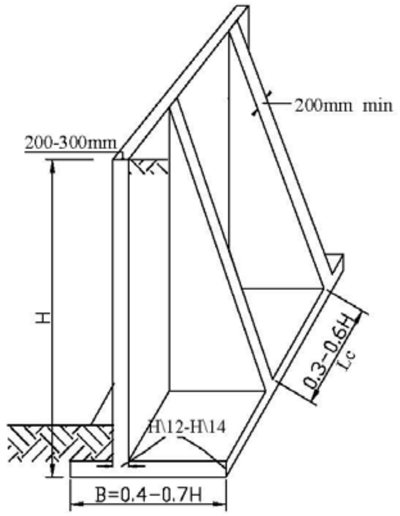
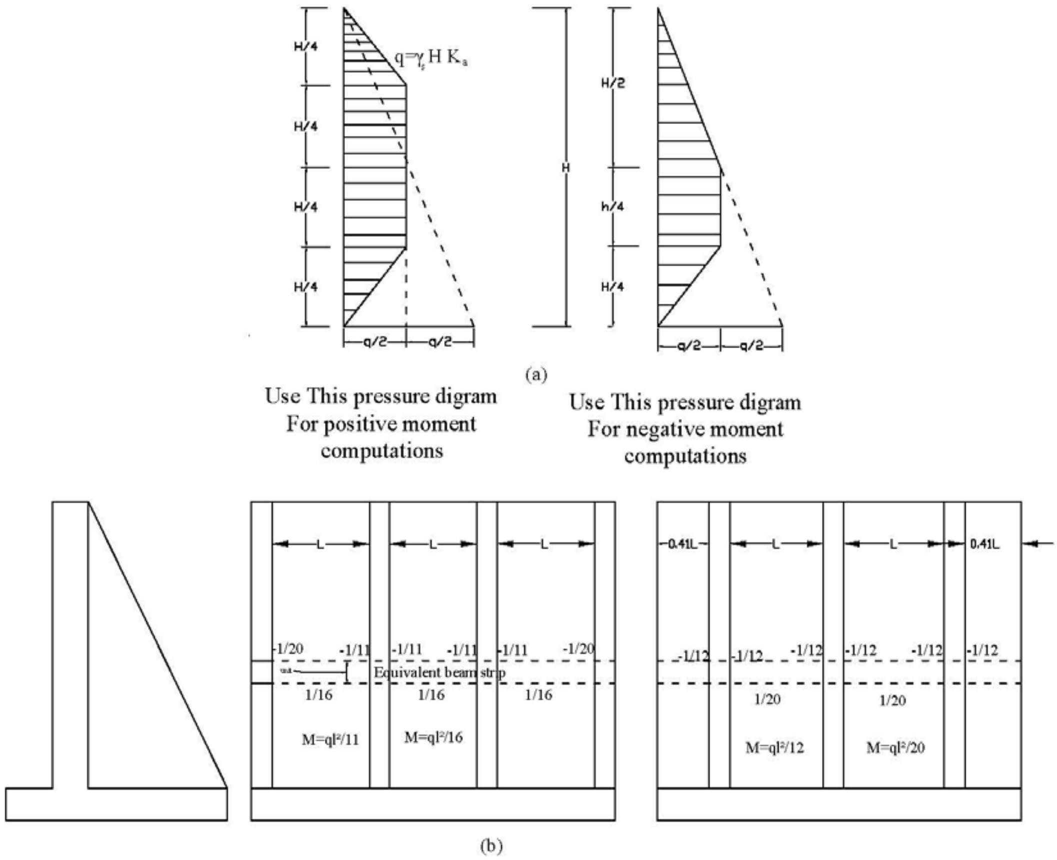


Fig.(2) Tentative design dimensions for a counterfort retaining wall.[6]



Use **q** from the shaded portions of the pressure diagrams in(a) . Moment coefficients are shown.

Fig. (3) Computation of bending moments in the horizontal direction for the counterfort stem[6]

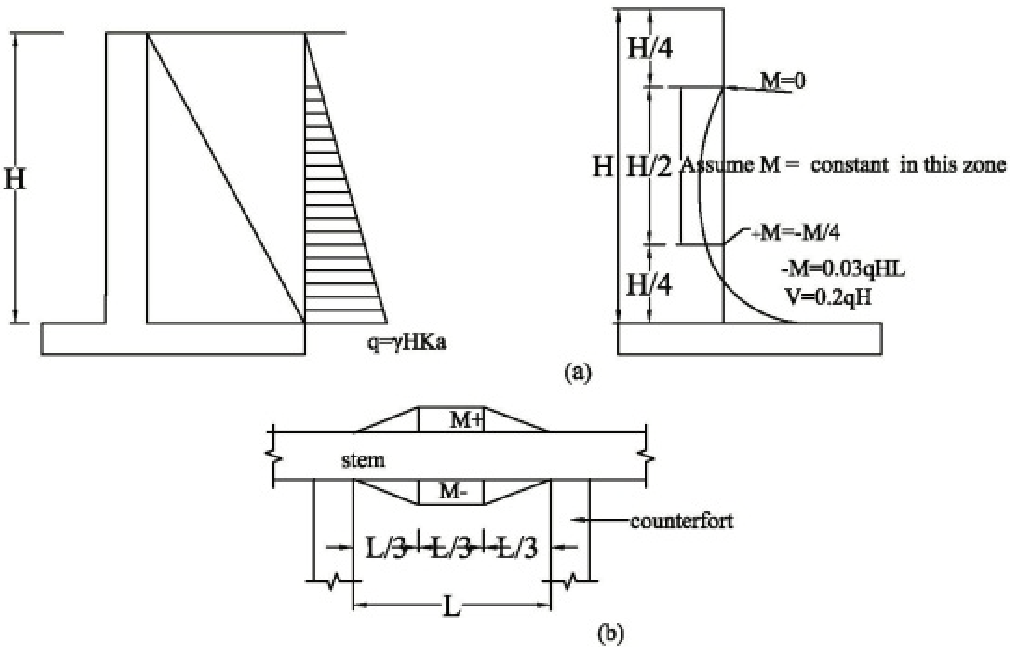


Fig.(4)Distribution of vertical moments in a counterfort wall stem for Huntigton's procedure [6]

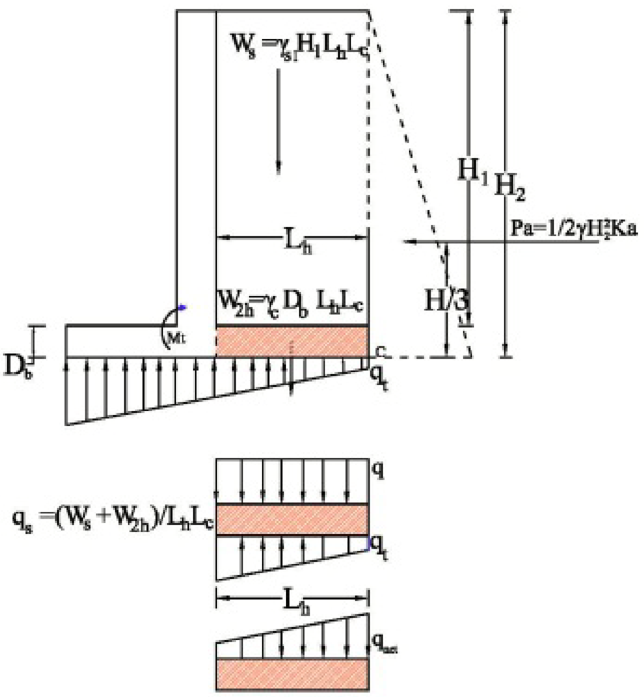


Fig.(5) Forces on the heel slab of a counterfort wall as proposed by Huntington [6]

where:
$P_r = F_r + P_p$,
$F_r = R\ \tan\delta_b + c_a B$
F_s= Factor of safety against sliding
P_{a2}=Total active pressure on the counterfort retaining wall
δ_b= Angle with the horizontal, made by the sloped backfill
R = the resultant of the vertical forces (concrete and soil)
c_a = (0.5 to 0.75) c, c cohesion of soil below the base

3. The location of the resultant R is within the middle one-third the full base width B:

From $\frac{R}{B\times L_c}\left(1\mp\frac{6e}{B}\right)\le q_a$ and $e\le\frac{B}{6}$

$\frac{3(M_r-M_o)}{BR}-1\ge 0$ --- (3)

where:
e=the eccentricity of R with the respect to the base.

4. Bearing Capacity:

$q_a-\frac{4R}{B\times L_c}+\frac{6(M_r-M_o)}{B^2\times L_c}\ge 0$ --- (4)

a) The ultimate resistance constraints

These constraints ensure the design to fit the strength requirements of the ACI code, i.e., any section must be strong enough to resist the applied forces. The applied forces involve moments and shear.

For ***the flexural constraints*** the moments of resistance per unit length at the critical sections should not be less than the values due to the factored loads. These are represented by:

Moment in toe part:

$\varphi M_{r,t}\ge M_{u,t}$---- (5) Positive moment in heel:

$\varphi M_{r,hp}\ge M_{u,hp}$ ---- (6) Negative moment in heel: $\varphi M_{r,hn}\ge M_{u,hn}$---- (7)

Positive moment in stem

$\varphi M_{r,sp}\ge M_{u,sp}$---- (8)

Negative moment in stem

$\varphi M_{r,sn}\ge M_{u,sn}$---- (9)

where $M_{u\ t}$, $M_{u\ hp}$, $M_{u\ hn}$, $M_{u\ sp}$ and $M_{u\ sn}$ are the ultimate bending moments per unit length, and φ is the strength reduction factor (=0.9).M_{rt}, M_{rhp}, M_{rhn}, M_{rstp} and M_{rsn} are the section moments capacities per unit length.

A limiting constraint is also employed to specify that the tension reinforcement at the section is not less than the minimum area (A_{smin}) and not greater than the maximum area (A_{smax}) required by the code. This constraint is applied to the various critical sections including:

a) At the toe (section dimensions are 1m ×D_b),

$A_{s1}\ge A_{s\ min}$---- (10)

$A_{smax}\ge A_{s1}$---- (11)

b) At the heel (section dimensions are 1m × D_b), for negative moment,

$A_{s5}\ge A_{s\ min}$ --- (12)

$A_{smax}\ge A_{s5}$---- (13)

c) At the heel (section dimensions are 1m × D_b), for negative moment in longitude direction ,

$A_{s6}\ge A_{s\ min}$ --- (14) $A_{smax}\ge A_{s6}$---- (15)

d) At the heel (section dimensions are 1m x D_b), for positive moment,

$A_{s8}\ge A_{s\ min}$---(16)$A_{smax}\ge A_{s8}$---(17)

e) At the stem (section dimensions are 1m × D_s), for horizontalreinforcement in each face.

$A_{s9}\ge A_{s\ min}$----- (18)

$A_{smax}\ge A_{s9}$----- (19)

$A_{s10}\ge A_{s\ min}$----- (20)

$A_{smax}\ge A_{s10}$----- (21)

f) At the stem (section dimensions are 1m× Ds), for vertical reinforcement in each face

$A_{s11}\ge A_{s\ min}$---- (22)

$A_{smax} \geq A_{s11}$---- (23)

$A_{s12} \geq A_{s\,min}$----- (24)

$A_{smax} \geq A_{s12}$---- (25)

whereD_s : thickness of stem

D_b : thickness of base

The reinforcement for the retaining wall is shown in Fig (1)

For ***the shear constraints,*** the section shear resistance should be greater than the applied shear force. This constraint is to be applied to the following sections:

a) At the toe (section dimensions are 1m × D_b),

$V_{c,t} \geq V_{u,t}$---- (26)

$$V_{u,t}=1.7\begin{bmatrix}(q_{toe}-\gamma_c \times D_b)\times L_t-0.5\times \\ \left(\frac{q_{toe}-q_{heel}}{B}\right)\times L_t^2\end{bmatrix} \text{-- (27)}$$

where:$q_{toe}=\frac{R}{B\times c}\left(1+\frac{6e}{B}\right)$, $q_{toe}=\frac{R}{B\times c}\left(1-\frac{6e}{B}\right)$

L_t: length of toe,L_h: length of heel.

b) At the heel (section dimensions are 1m x D_b),

$V_{c,h1} \geq V_{u,h1}$---- (28)$V_{c,h2} \geq V_{u,h2}$---- (29)

where:

$$V_{u,h1}=1.7\begin{bmatrix}(\gamma_c D_b+\gamma_{s1} H_1-q_{heel})L_h- \\ \frac{1}{2B}(q_{toe}-q_{heel})L_h^2\end{bmatrix},$$

$$V_{u,h2}=1.7\left[\left((\gamma_c D_b+\gamma_{s1} H_1)-q_{heel}\right)L_c\right]$$

H_1: height of stem wall,L_h: length of heel

c) At the stem (section dimensions are 1m × D_s)

$V_{c,m} \geq V_{u,m}$-- (30)$V_{c,m} = 0.2K_a\gamma_{s1}H_1^2L_c$---- (31)

where

$\emptyset V_{ci}=\frac{\emptyset}{6}\sqrt{f_c'}$for all sections and $\emptyset$=is the strength reduction factor (=0.85)

4. <u>Objective Function</u>

The statement of the problem is as follows:

Minimize C(X) subject to the inequality constraints:

$g_j(x) \geq 0$; j=1,2,…………,m --- (32)where X is the vector of independent design variables and C(X) is the objective function.

In the present study, the objective function is defined as the total cost of counterfort retaining wall (material & labor). This includes the followings:

1- Cost of concrete including cost of materials, mixing, placing and curing.

2- Cost of various steel reinforcement. This cost includes the material and labor costs.

3- Cost of formwork.

Therefore, the cost of the counterfort retaining wall is equal to the summation of costs of the wall, the base, and the counterforts. These are given by:

For base

Cost of the concrete :

$C_{cb}=D_b\times B \times L_c \times R_1$----- (33)

Cost of the reinforcement:

$C_{rb}= (A_{stoe}+A_{s\,heel}) \times\gamma_{steel}$---- (34)

Cost of the formwork:

$C_{fb}=2 \times D_b \times L_c \times R_2$--- (35)

where:

$A_{stoe} = (A_{s1}+A_{s2}+A_{s3}+A_{s4}) \times (L_t+D_s) \times L_c$.

$A_{sheel}= (A_{s5}+A_{s6}+A_{s7}+A_{s8}) \times (L_h) \times L_c$.

R_1: the ratio of cost of the concrete per cubic meter to cost of thereinforcement per Newton

R_2: the ratio of cost of the formwork per square meter to thecost of the reinforcement perNewton

γ_{steel} =unit weight of steel

For the stem

Cost of the concrete:

$C_{cs}= H_1 \times D_s \times L_c \times R_1$ ---- (36)

Cost of the reinforcement:

$C_{rc}= A_{s\ stem}\times \gamma_{steel}$----- (37)

Cost of the formwork:

$C_{fc} = (2\times H_1 \times L_c - D_c \times H_1)\times R_2$ ---- (38)

where:

$A_{s\ stem} = (A_{s9} + A_{s10} + A_{s11} + A_{s12})\times H_1 \times L_c$.

For counterfort

Cost of the concrete:

$C_{cc} = \frac{1}{2}\times L_h \times D_c \times H_1 \times R_1$ ----- (39)

Cost of the reinforcement:

$C_{rc} = A_{s\ counterfort}\ \gamma_{steel}$ --- (40)

Cost of the formwork:

$C_{fc} = \left(H_1 \times L_h + D_c \times \sqrt{H_1^{\ 2} + L^2{}_h} \right)\times R_2$ - (41)

where:

$$A_{s\ counterfort} = \begin{pmatrix} A_{s13} \times (\sqrt{H_1^{\ 2} + L_h^{\ 2}}) + \\ (A_{s14} + A_{s15})\times (H_1 \times L_h) \end{pmatrix}$$

Thus, the objective function or the total cost, C, is expressed mathematically as:

$$C(x) = [C_{cb} + C_{cs} + C_{cc}] + [C_{rb} + C_{rs} + C_{rc}] + [C_{fb} + C_{fs} + C_{fc}]/L_c$$

----(42)

Solution Procedure

The optimization problem formulated in the previous section is a constrained non-linear programming problem. Such problem can be solved by the interior penalty function method using sequential unconstrained minimization technique. Method of Hooke and Jeeves(as cited in Ref.8)method is employed to find the search direction.

In the penalty function methodsit is to transform the problem into a sequence of unconstrained minimization problems[7and 8].

Z=C(X) +P(X)

where P(X) is the penalty function

where $P(X) = r\sum_{j=1}^{m} \frac{1}{g_j(X)}$

r_k is positive. The function $Z = \phi(X, r)$ then takes the form

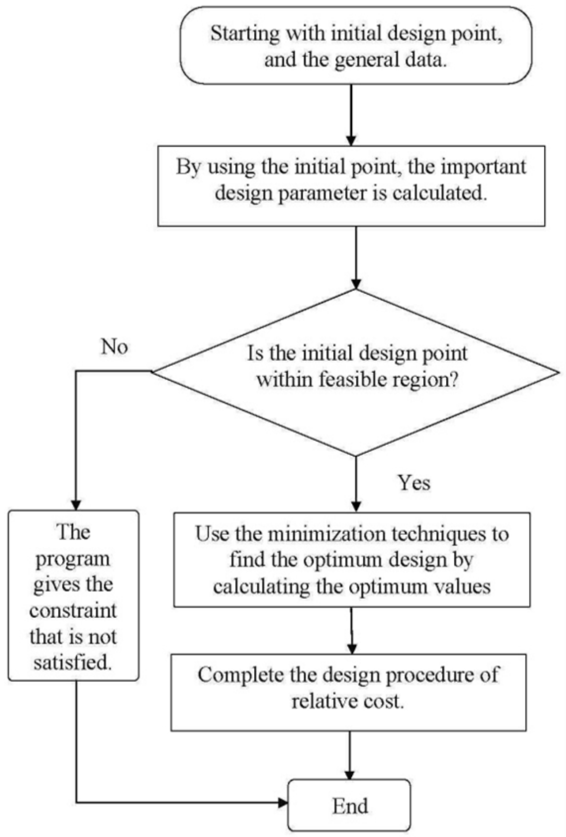
$$Z = \varphi(X, r) = C(X) + r_k \sum_{j=1}^{m} \frac{1}{g_j(X)} \qquad (43)$$

The flow chart for the generated computer program based on the chosen method of solution is depicted in Fig.(6)

Results and Discussions

The objective of the present study is to obtain the minimum cost design, therefore many applications have been considered to well understand the problem.

These applications involve solving many numerical examples in order to illustrate the



effects of various design variables and different parameters on the optimal design Finally the minimum relative cost of the counterfort retaining wall for one meter length is given.

For basic values ofrequired parameters are taken as bearing capacity of soil q_{al1} =120 kN/m^2; yield stress of steel f_y=415 MPa; concrete cylinder compressive strength f'_c=21 MPa; thickness of counterfort =0.5 m; unit weight of reinforced concrete γ_c=24 kN/m^3; unit weight of soil (backfill) γ_{s1} =20 kN/m^3 ;unit weight of soil under base γ_{s2} = 17.950 kN/m^3;cohesive strength of soil c_2 =19.12 kN/m^2 ;angle of internal friction (for backfill soil) $ø_1$=30; angle of internal friction (for base soil) $ø_2$ =28; cost of steelC_s=1000000 I.D/ton ;cost of concrete C_c=150000 I.D /m^3;cost of formworkC_f=7500 I.D /m^2. The first counterfort is started a distance 0.5 L_c from the end of the wall,and Fig. (1) is used. These values are only used as a guide to starting with initial design point.

1. Effect of Total Height of Wall

Table (1) gives the optimum distanceL_c between counterforts.$L_c$increases as the total height H_2 increases. The data from these Table also leads to a relationship between the optimum L_c and the total height H_2. It can be said that the optimum L_c equals about (0.3 to 0.36) of H_2. This relation is not unique but it usually depends on many factors like bearing capacity and material properties.

The total height of counterfort retaining wall has an effect on the optimum stem thickness which increases with the increase of total height H_2. Also the thickness of the base increases as the wall height increases. The stem and base steel reinforcement relates to the total height H_2 of counterfort retaining wall in that it increases with increasing the total height.

The relationship between the total cost of wall and the total height H_2 is approximately linear as shown in Fig. (7).

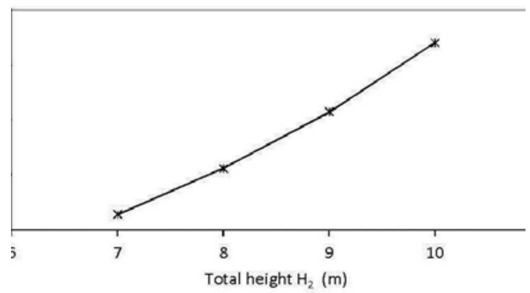
2. Effectof soil Bearing Capacity

From the Table (2), it is clear that when the bearing capacity reduces, the optimum distance between counterforts increases. The length of the base B increases and thickness of the base D_b decreases as the bearing capacity decreases. The stem thickness D_s seems not to change as the bearing capacity reduces.

Table (1) Optimum spacing of counterforts for various total height values of counterfort retaining wall

H_2 m	optimum L_C m	Relative cost (N/m)	H_1 m	L_t m	D_S m	L_h m	B m	D_b m	A_{S1} mm^2	A_{S2} mm^2	A_{S3} mm_2	A_{S4} mm^2	A_{S5} mm^2	A_{S6} mm^2	A_{S7} mm^2	A_{S8} mm^2	A_{S9} mm^2	A_{S10} mm^2	A_{S11} mm^2	A_{S12} mm^2	A_{S13} mm^2	A_{S14} mm^2	A_{S15} mm^2
7	2.079	11367	6.472	2.827	.246	.777	3.85	.528	3491	856	856	856	1443	856	856	856	293	293	568	293	4051	1338	368
8	2.631	15522	7.352	3.386	.289	0.997	4.671	.648	3935	1097	1097	10974	1850	1097	1097	1097	377	382	718	377	5833	1716	529
9	3.067	20654	8.229	4.043	.336	1.144	5.523	.771	4580	1343	1343	1343	2264	1343	1343	1343	473	473	834	473	8106	2100	690
10	3.571	26974	9.081	4.660	.388	1.400	6.447	.919	4996	1639	1639	1639	2764	1639	1639	1639	576	576	970	576	10226	2436	886

B.C=120 kN/m^2 γ_{s1}=20 kN/m^3 f'_c=21 MPa f_y=415 MPa



tionship between relative cost and total height of

Retaining wall

Table (2) The optimum design for wall total height with different bearing capacity values

(for (γ_{s1}=20 kN/m^3 f'_c=21 MPa f_y=415 MPa)

H_2 m	B.C kN/m^2	L_C m	Relative cost (N/m)	H_1 m	L_t m	D_S m	L_h m	B m	D_b m	A_{S1} mm^2	A_{s4} mm^2	A_{S5} mm^2	A_{S6} mm^2	A_{S7} mm^2	A_{S8} mm^2	A_{S9} mm^2	A_{S10} mm^2	A_{S11} mm^2	A_{S12} mm^2	A_{S13} mm^2	A_{S14} mm^2	A_{S15} mm^2
7	60	2.475	12470	6.513	3.704	.248	.995	4.947	.487	3891	775	1306	775	775	775	297	384	682	297	3987	1053	441
	80	2.439	11959	6.505	3.364	.248	.877	4.488	.496	3864	791	1334	791	791	791	296	373	673	296	4430	1237	433
	100	2.326	11623	6.483	3.028	.247	.863	4.183	.517	3552	835	1408	835	835	835	294	339	639	294	4236	1305	412
	120	2.079	11367	6.472	2.827	.246	.777	3.85	.528	3491	856	1443	856	856	856	293	293	568	293	4051	1338	368
8	60	2.809	17108	7.403	4.468	.291	1.192	5.951	.597	4430	995	1677	995	995	995	383	433	769	383	5355	1271	568
	80	2.778	16394	7.390	4.042	.291	1.077	5.41	.608	4373	1017	1715	1017	1017	1017	382	423	760	382	5815	1467	560
	100	2.762	15886	7.375	3.69	.290	1.017	4.997	.625	4195	1051	1773	1051	1051	1051	380	420	755	380	6076	1644	557
	120	2.631	15522	7.352	3.386	.289	0.997	4.671	.648	3935	1097	1850	1097	1097	1097	377	382	718	377	5833	1716	529
9	60	3.206	22855	8.276	5.231	.339	1.43	7.000	.725	4888	1248	2105	1248	1248	1248	478	502	872	478	6957	1506	725
	80	3.185	21884	8.268	4.775	.338	1.273	6.391	.732	4918	1265	2133	1265	1265	1265	478	497	867	478	7704	1755	719
	100	3.145	21204	8.254	4.410	.338	1.18	5.930	.746	4829	1293	2180	1293	1293	1293	476	485	856	476	8147	1955	709
	120	3.067	20654	8.229	4.043	.336	1.144	5.523	.771	4580	1343	2264	1343	1343	1343	473	473	834	473	8106	2100	690

Also thistableshows that steel areassuch as (A_{s2} to A_{s8}, and A_{s14}) decrease while the area (A_{s1}, A_{s10}, A_{s11}, and A_{s15}) increases when the bearing capacity reduces. Steel areas (A_{s9} and A_{s12}) seem not to alter as the bearing capacity of soil varies. In addition, the reduction of bearing capacity leads to increase the total cost.

3. Effect of materials properties

Table (3) reveals that the compressive strength of concrete has an effect on the optimum distance between counterforts L_c. The increase in compressive strength of concrete reduces L_c. Increasing the concrete compressive strength leads to a reduction in the base and stem thickness, consequently different steel areas are needed.

A_{s2} to A_{s9}, As_{12}, A_{S13}, A_{s14} and, A_{s15} decease as f'_c increases. A_{s1}, A_{s10}, and A_{s11} increase with increasing f'_c. The increase in area of steel may be attributed to the reductionin the thickness of both base and stem.

In addition, increasing the compressive strength of concrete leads to a reduction in the relative cost of the wall.Finally, according to the above results, it may be said that the optimum design is achieved by using concrete of high strength (keeping in mind that the cost of concrete is considered here as constant irrespective of its strength).

The effect of yield strength of steel is shown in Table (4). Results reveal that the increase of steel strength increases the optimum distance between counterforts L_c while no effect is noticed on the stem thickness D_s. The increase in yield strength of steel has a little effect on the base thickness.

The effect of increasing the yield strength of steel is to reduce steel areas in optimum section as it is clear from Table (4). Therefore, these results indicate that it is economical to use steel of high strength in design.

Concerning the effect of backfill soil, the results obtained by varying the backfill density are shown in Table (5). Increasing soil density seems to have very little effect on optimum distance between counterforts as illustrated in these Tables (with the range of γ_{s1} considered in this study).Alsoincreasing soil density causes the base and the stem thickness , base width, all areas of steel toincrease.

Proportions of counterfort retaining wall

Dimensions of counterfort retaining wall should be adequate for structural stability and to satisfy design requirements. The tentative dimensions shown in Fig. (1) is based in part on history of satisfactorily constructed walls, and may be used in the absence of other data, but in an overly conservative design.[6]

For the initial point required by the generated program, the dimensions of the wall were selected within the values given in Fig. (1). Then, and according to the parameters used, the program gives the optimum design including the optimum dimensions of the wall.

Table (6) shows a comparison between the optimum dimensions obtained in this study and the values used as the initial point which is suggested in Fig. (2). Theother values of H_2(7, 8, and 9) m the same analysis is conducted, and it is found that:

Table (3) The optimum design for different values of concrete cylinder compressive strength values (21,30,40,50) MPa, for ($B.C=120\ kN/m^2$ $\gamma_{s1}=20\ kN/m^3$ $fy=415\ MPa$)

H_2 m	f'_c MPa	L_C m	Relative cost (N/m)	H_1 m	L_t m	D_S m	L_h m	B m	D_b m	A_{S1} mm^2	A_{S2},A_{s3} A_{s4} mm^2	A_{S5} mm^2	A_{S6} mm^2	A_{S7} mm^2	A_{S8} mm^2	A_{S9} mm^2	A_{S10} mm^2	A_{S11} mm^2	A_{S12} mm^2	A_{S13} mm^2	A_{S14} mm^2	A_{S15} mm^2
7	**21**	2.079	11367	6.472	2.827	.246	.777	3.85	.528	3491	856	1443	856	856	856	293	293	568	293	4051	1338	368
	30	2.083	10893	6.538	2.790	.225	.814	3.828	.462	4070	725	1223	725	725	725	250	321	680	250	4113	1342	372
	40	2	10639	6.584	2.77	.210	.829	3.809	.416	4647	633	1066	633	633	633	231	338	752	220	3974	1294	360
	50	1.923	10521	6.614	2.739	.199	.854	3.8	.386	5085	572	1077	572	572	572	238	347	806	198	3777	1237	348
8	**21**	2.631	15522	7.352	3.386	.289	0.997	4.671	.648	3935	1097	1850	1097	1097	1097	377	382	718	377	5833	1716	529
	30	2.577	14773	7.44	3.800	.262	1 .00	4.641	.559	4722	918	1550	918	918	918	323	431	838	323	5906	1714	524
	40	2.475	14334	7.490	3.260	.242	1.100	4.601	.510	5093	820	1383	820	820	820	310	455	928	284	5300	1596	507
	50	2.304	14098	7.531	3.24	.228	1.11	4.578	.469	5645	738	1359	738	738	738	298	483	963	257	4983	1490	474
9	**21**	3.067	20654	8.229	4.043	.336	1.144	5.523	.771	4580	1343	2264	1343	1343	1343	473	473	834	473	8106	2100	690
	30	2.941	19528	8.327	3.943	.302	1.224	5.47	.674	5244	1147	1935	1147	1147	1147	405	501	953	405	7529	1995	669
	40	2.762	18830	8.4	3.9	.278	1.125	5.429	.601	5965	1003	1692	1003	1003	1003	357	505	1031	357	7125	1884	634
	50	2.551	18416	8.443	3.82	.261	1.31	5.391	.556	6433	913	1562	913	913	913	327	477	1061	323	6414	1710	588

Table (4) The optimum design different values of yield strength of steel , for (γ_{s1}=20 kN/m^3 B.C =120 kN/m^2 f'_c=21 MPa)

H_2 m	f_y MPa	L_C m	Relative cost (N/m)	H_1 m	L_t m	D_S m	L_h m	B m	D_b m	A_{S1} mm^2	A_{S2},A_{s3} A_{s4} mm^2	A_{S5} mm^2	A_{S6} mm^2	A_{S7} mm^2	A_{S8} mm^2	A_{S9} mm^2	A_{S10} mm^2	A_{S11} mm^2	A_{S12} mm^2	A_{S13} mm^2	A_{S14} mm^2	A_{S15} mm^2
7	250	1.984	13088	6.454	2.70	.245	.900	3.846	.546	5218	892	2497	892	892	892	291	409	898	291	5641	1972	581
	350	2.049	11865	6.475	2.848	.246	.760	3.859	.525	4207	850	1699	850	850	850	293	311	675	293	4962	1575	429
	415	2.079	11367	6.472	2.827	.246	.777	3.85	.528	3491	856	1443	856	856	856	293	293	568	293	4051	1338	368
	460	2.203	10916	6.462	2.759	.246	.847	3.851	.538	2972	720	1333	720	720	720	240	275	545	240	3638	1236	350
8	250	2.092	18081	7.371	3.526	.290	.856	4.672	.628	7166	1056	2957	1056	1056	1056	380	396	938	380	8824	2391	699
	350	2.500	16205	7.355	3.406	.289	.971	4.665	.645	4736	1091	2181	1091	1091	1091	377	406	807	377	6722	1954	596
	415	2.631	15522	7.352	3.386	.289	0.997	4.671	.648	3935	1097	1850	1097	1097	1097	377	382	718	377	5833	1716	529
	460	2.857	14908	7.335	3.251	.288	1.123	4.662	.664	3277	928	1717	928	928	928	309	409	706	309	5088	1593	516
9	250	2.604	23776	8.199	3.835	.335	1.337	5.507	.801	6810	1402	3923	1402	1402	1402	470	552	1168	470	9630	2687	968
	350	2.873	21581	8.228	4.043	.336	1.141	5.521	.772	5425	1344	2687	1344	1344	1344	473	480	923	473	9866	2325	765
	415	3.067	20654	8.229	4.043	.336	1.144	5.523	.771	4580	1343	2264	1343	1343	1343	473	473	834	473	8106	2100	690
	460	3.165	19757	8.222	3.992	.335	1.194	5.523	.778	4021	1115	2064	1115	1115	1115	387	444	777	387	7234	1913	640

Table (5) The optimum design differentvalues of density of soil *(16,18,20)*kN/m^3, for (*B.C= 120 kN/m^2 f'_c=21 MPaf_y=415 MPa)*

H_2 *m*	γ_{s1} *kN/m^3*	L_C *m*	Relative cost *(N/m)*	H_1 *m*	L_t *m*	D_S *m*	L_h *m*	B *m*	D_b *m*	A_{S1} *mm^2*	A_{S2},A_{s3} A_{s4}*mm^2*	A_{S5} *mm^2*	A_{S6} *mm^2*	A_{S7} *mm^2*	A_{S8} *mm^2*	A_{S9} *mm^2*	A_{S10} *mm^2*	A_{S11} *mm^2*	A_{S12} *mm^2*	A_{S13} *mm^2*	A_{S14} *mm^2*	A_{S15} mm^2
7	**16**	2.083	9998	6.523	2.517	.219	.779	3.515	.477	3137	754	1271	754	754	754	238	270	576	238	3467	1179	297
	18	2.083	10679	6.497	2.675	.233	.779	3.686	.503	3319	806	1359	806	806	806	266	271	572	266	3818	1261	333
	20	2.079	11367	6.472	2.827	.246	.777	3.85	.528	3491	856	1443	856	856	856	293	293	568	293	4051	1338	368
8	**16**	2.525	13506	7.427	3.064	.254	.940	4.258	.573	3705	947	1597	947	947	947	308	349	695	308	4967	1482	410
	18	2.488	14512	7.396	3.279	.272	0.918	4.469	.604	3966	1008	1701	1009	1009	1009	344	344	680	344	5503	1577	452
	20	2.631	15522	7.352	3.386	.289	0.997	4.671	.648	3935	1097	1850	1097	1097	1097	377	382	718	377	5833	1716	529
9	**16**	3.049	17865	8.311	3.590	.293	1.171	5.053	.668	4125	1177	1985	1177	1177	1177	386	455	838	386	6610	1794	553
	18	3.067	19257	8.272	3.84	.315	1.141	5.296	.728	4406	1257	2119	1263	1257	1257	430	460	838	430	7498	1965	624
	20	3.067	20654	8.229	4.043	.336	1.144	5.523	.771	4580	1343	2264	1343	1343	1343	473	473	834	473	8106	2100	690

Table (6) Optimum dimensions for section of counterfort retaining wall with different parameters **for** $H_2=7m$

Dimensions suggested in Ref.[27]	Bearing apacity(kN/m^2) $\gamma_{s1}=20\ kN/m^3 f'_c=21\ MPa f_y=415\ MPa$				Bearing apacity(kN/m^2) $\gamma_{s1}=18\ kN/m^3 f'_c=21\ MPa f_y=415\ MPa$				Bearingcapacity(kN/m^2) $\gamma_{s1}=16\ kN/m^3 f'_c=21\ MPa f_y=415\ MPa$				Compressive strength of concrete MPa $\gamma_{s1}=20\ kN/m^3 f_y=415\ MPa$				Yield strength of steel MPa $\gamma_{s1}=20\ kN/m^3 f'_c=21\ MPa$			
	60	80	100	120	60	80	100	120	60	80	100	120	21	30	40	50	250	350	415	460
L_C $(0.3\text{-}0.6)H_2$	.354	.348	.332	.297	.359	.355	.331	.298	.361	.357	.323	.298	.297	.298	.286	.275	.2834	.2927	.297	.3147
B $(0.4\text{-}0.7)H_2$	.707	.641	.598	.550	.681	0.615	0.565	.527	.654	.589	.541	.502	.550	.5469	.5441	.5429	.5494	.5513	.550	.550
D_b $(.083\text{-}.071)H_2$	.0696	.0709	.0739	.0754	.0659	.068	.0700	.0719	.06	.063	.0657	.0681	.0754	.0660	.0594	.0551	.0780	.0754	.0754	.0769
D_S $(.083\text{-}.071)H_2$	.0354	.0354	.0354	.0354	.0334	.0334	.0334	.0334	.032	.0316	.0316	.0316	.0351	.0321	.030	.0284	0351	.0351	.0351	.0351

1. The distance between counterforts is from 0.214 to 0.366 of the wall height H_2.
2. The width of the base is from 0.5 to 0.78 of the wall height H_2. The value 0.78 H_2 appeared where the bearing capacity of soil is less than 80 kN/m^2
3. The thickness of the baseis from 0.055 to 0.0941of the wall height H_2.
4. The thickness of the wall is from 0.0284 to 0.0377 of the height H_2 and it is less than half thickness of the base.

<u>Conclusions</u>

Based on the results obtained in this study the following conclusions may be drawn:

1- The optimum distance between counterforts is equal to (0.275 to 0.366) of the height of wall H_2, and on increasing the price of concrete this percentage decreases to 0.214 H_2

2- The total cost of counterfort retaining wall linearly increases with increasing the total height (H_2)

3- Reduction of bearing capacity of soil leads to increasing the length of the base and decreasing the thickness of the base while the thickness of the stem is not affected.

4- The relative cost of wall increases as the bearing capacity of soil decreases.

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